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Degenerate electron gas and star stabilization in D dimensions

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Summary. — The quantum-mechanical energy, the Fermi energy, and the degeneracy pressure are derived for a degenerate Fermi gas in D spatial dimensions in the non-relativistic and the extreme relativistic cases. The stability of the system due to the balancing between degeneracy pressure forces and gravitational forces is investigated. We also examine the asymptotic behavior of our results in very large dimensions and their limits in the infinite-dimensional space. For computation purposes, we apply these findings to white-dwarf and neutron stars.

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1. – Introduction

It is widely believed that the space dimension D plays an important role in determining the behavior of a physical system. Besides its mathematical interest [1, 2], the D-dimensional space has been used in the study of Schrödinger equation with different kinds of potentials [3-7] and in quantum field theories [8, 9]. Some workers investigated higher-dimensional gravity [10, 11] and others examined rotating black holes in multi-dimensional space [12-15]. Recently, considerations of gravitational collapse in higher dimensions have been reported [16-20]. Furthermore, the stability of a white-dwarf star has been investigated in three-dimensional space [21]. The role of the electron degeneracy pressure in the discussion of the formation of white dwarfs and neutron stars has been emphasized by many authors [22-26].

The purpose of the present paper is to examine the effect of the space dimension, D, on the behavior of some of the properties of degenerate electrons or neutrons in a star. The interest of the present author in the role of space dimension in white-dwarf stars,

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besides its mathematical interest, is simulated by the wide belief that most stars end as white dwarfs, and thus they have impact on the history of the Universe. Because during the formation of a white dwarf, mass is lost to the surrounding interstellar medium. This, therefore, provides insights into the mass content of a galaxy. In addition, white dwarfs have magnetic fields and thus they behave as pulsators and emit spectra in a varying way. The structure of this paper is as follows: in sect. **2**, we outline the degeneracy pressure and total energy of the system. Section **3** deals with extreme relativistic degenerate gas. In sect. **4**, we consider neutron degenerate gas. Conclusions and results are given in sect. **5**.

2. – Degeneracy pressure and total energy of the system

Consider a large number of identical, non-interacting, spin-(1/2) particles in a D-dimensional box with impenetrable walls each of length L. Because electrons are fermions with spin (1/2), each spatial orbital has S possible spin states. As was shown by Menon and Agrawal [27], S must be $(D-1)$ for $D>1$. Denoting by N_S the number of particle states having energies $E=\hbar^2k^2/2m$, these states are contained within a hyper-sphere of radius k in k-space. Therefore,

$$N_S=(D-1)\left(\frac{L}{2\pi}\right)^D V_D, \tag{1}$$

where V_D is the volume of the hyper-sphere which is given by [28]

$$V_D=\frac{\pi^{D/2}k^D}{\Gamma(1+D/2)}, \tag{2}$$

where $\Gamma(x)$ is the gamma-function. Therefore,

$$N_S=(D-1)(L/2\pi)^D\frac{\pi^{D/2}k^D}{\Gamma(1+D/2)}. \tag{3}$$

For the non-relativistic case, $E=\hbar^2k^2/2m$ and thus eq. (3) becomes

$$N_S=(D-1)(L/2\pi)^D\frac{\pi^{D/2}}{\Gamma(1+D/2)}\left(\frac{2m}{\hbar^2}\right)^{D/2}E^{D/2}. \tag{4}$$

The density of states, $g(E)$, is defined as the number of particle quantum states per unit energy range, so that

$$g(E)=\frac{\mathrm{d}N_S}{\mathrm{d}E}=\frac{D}{2}(D-1)(L/2\pi)^D\frac{\pi^{D/2}}{\Gamma(1+D/2)}\left(\frac{2m}{\hbar^2}\right)^{D/2}E^{(D-2)/2}. \tag{5}$$

The Fermi energy is obtained by requiring that the total number of particles (Nq) in the system is

$$Nq=\int_0^{E_\mathrm{F}}g(E)\mathrm{d}E,$$

with q being the number of electrons per nucleon. Using $L^3 = V$, the above requirement yields

$$E_{\mathrm{F}} = \frac{2\pi\hbar^2}{m}\left[\frac{Nq}{V}\,\frac{\Gamma(1+D/2)}{D-1}\right]^{2/D}. \tag{6}$$

The quantum-mechanical energy E_m of a Fermi gas in the ground state (at absolute temperature $T = 0$) is $E_m = \int_0^{E_{\mathrm{F}}} E\, g(E)\mathrm{d}E$, which upon using eq. (5) yields

$$E_m = \frac{D}{D+2}\left(\frac{2\pi\hbar^2}{m}\right)\left[\frac{\Gamma(1+D/2)}{D-1}\right]^{2/D}[Nq]^{(D+2)/D}\,V^{-2/D}. \tag{7}$$

It can be easily checked that the above results for $g(E)$, E_{F}, and E_m reduce to the well-known expressions in the three-dimensional space ($D = 3$), with the result [29]

$$g(E) = \frac{L^3}{2\pi^2}(2m/\hbar^2)^{3/2}E^{1/2}, \tag{5a}$$

$$E_{\mathrm{F}} = \frac{\hbar^2}{2m}\left(3\pi^2\frac{Nq}{V}\right)^{2/3}, \tag{6a}$$

$$E_m = \frac{\hbar^2}{10m\pi^2}\left(3\pi^2 Nq\right)^{5/3}V^{-2/3}. \tag{7a}$$

In terms of the Fermi energy, eq. (7) can be written as

$$E_m = \frac{D}{D+2}NqE_{\mathrm{F}}. \tag{8}$$

This quantum-mechanical energy plays a role that is analogous to the internal thermal energy of an ordinary gas. In particular, it exerts a pressure on the walls so that when the system expands by an amount $\mathrm{d}V$, the mechanical energy decreases by $\mathrm{d}E_m = -\frac{2}{D}\frac{E_m}{V}\mathrm{d}V$, and this shows up as work done on the outside, $\mathrm{d}W = P\mathrm{d}V$. Therefore, the quantum pressure P (called degeneracy pressure) is

$$P = \frac{2}{D}\frac{E_m}{V} = \frac{4\pi\,\hbar^2}{m(D+2)}\left[\frac{\Gamma(1+D/2)}{D-1}\right]^{2/D}\rho^{(D+2)/D}, \tag{9}$$

where $\rho = Nq/V$ is the particle's density. The above result yields the three-dimensional case, namely [29]

$$P = \frac{\hbar^2}{5m}(3\pi^2)^{2/3}\rho^{5/3}. \tag{9a}$$

It must be noted that this degeneracy pressure has nothing to do with electron-electron interaction or thermal motion that we excluded here. This pressure is strictly quantum mechanical, which is due to the anti-symmetrization requirement for the wave function of identical fermions. An interesting illustration of the role of the degeneracy pressure is the stabilization of a star against gravitational collapse. At some stage, a star (like a white dwarf) is stabilized against gravitational collapse by the pressure of the degenerate electrons in the interior. This occurs at a star radius R at which the total energy (quantum-mechanical energy and the gravitational energy) is a minimum. To that end, we calculate the gravitational energy E_g of a uniform hyper-sphere of radius R and mass M in D-dimensional space.

Considering building up a hyper-sphere by layers, when its mass is m and its radius is r, the work necessary to bring in the next layer is $\mathrm{d}W = -Gm\mathrm{d}m/r^{D-2}$, where G is the universal gravitational constant. Writing $\mathrm{d}m = \rho \mathrm{d}V$, and using eq. (2), we get

$$\mathrm{d}W = -\frac{G\pi^D D}{[\Gamma(1+D/2)]^2}\rho^2 r^{D+1}\mathrm{d}r.$$

The total gravitational energy of the hyper-sphere is thus

$$E_g = \int_0^R \mathrm{d}W = -\frac{GD\pi^D R^{D+2}\rho^2}{(D+2)\left[\Gamma\left(1+\frac{D}{2}\right)\right]^2},$$

which yields, using $\rho = NM/V$ and using eq. (2),

$$E_g = -G\frac{D}{D+2}\frac{N^2M^2}{R^{(D-2)}}, \tag{10}$$

where N is number of nucleons and M is the nucleon mass. The substitution of eq. (2) into eq. (7) and with the help of eq. (10) enables us to write the total energy $E(= E_g + E_m)$ as

$$E = \frac{A}{R^2} - \frac{B}{R^{(D-2)}}, \tag{11}$$

where

$$A = \frac{D}{D+2}\left(\frac{2\hbar^2}{m}\right)\frac{(Nq)^{(D+2)/D}}{(D-1)^{2/D}}\left[\Gamma\left(1+\frac{D}{2}\right)\right]^{4/D}, \tag{12}$$

$$B = G\frac{D}{(D+2)}N^2M^2. \tag{13}$$

Requiring the total energy to be a minimum implies that $\mathrm{d}E/\mathrm{d}R = 0$, which gives $R^{(D-4)} = B(D-2)/2A$. The substitution of eqs. (12) and (13) yields

$$R^{(D-4)} = \frac{(D-2)(D-1)^{2/D}}{4(\hbar^2/GmM^2)}\frac{1}{\left[\Gamma\left(1+\frac{D}{2}\right)\right]^{4/D}}\frac{N^{(D-2)/D}}{q^{(D+2)/D}}. \tag{14}$$

The above equation clearly shows the dependence of the radius of stability of the star on the space dimension D and it easily reduces to the known formula in the three-dimensional space ($D = 3$) [29], namely $R = \frac{\hbar^2}{GnM^2}(\frac{9\pi}{4})^{2/3}\frac{q^{5/3}}{N^{1/3}}$. To demonstrate the role of the space dimension D on the radius of stabilization of a star, we consider a star whose mass equals the mass of the Sun (1.989×10^{30} kg) that gives the number of

TABLE I. – *Stability radius R for different values of the space dimension, D.*

D	R(m)	D	R(m)
3	7.157×10^6	10	2.046×10^3
4	-----------	20	38.417
5	3.546×10^8	40	5.972
6	1.497×10^6	60	3.809
7	1.039×10^5	100	2.018
8	1.845×10^4	10^4	1.0066
9	5.310×10^3	10^5	1.00063

nucleons $N = 1.188 \times 10^{57}$. Using $q = 1/2$ and $\hbar^2/(GmM^2) = 6.534 \times 10^{25}$ enables us to calculate R for different values of D, which we show in table I.

We note that R has its largest value when $D = 5$ and beyond that it decreases as D increases. The present author believes that this maximum value of R at $D = 5$ has no physical grounds, because if one calculates the Fermi energy of electrons for the white dwarf, he will conclude that the Fermi energy is in the order of the rest mass energy. This implies that the system is getting dangerously relativistic, and thus an appropriate relativistic treatment of the system must be considered, which we carry out in sect. **3**. For very large D, the value of R approaches unity. This can be shown by using Stirling's asymptotic formula $\Gamma(1+n) \approx \sqrt{2\pi} n^{n+1/2} \exp[-n]$, with $n = D/2$ and one gets $R^{D-4} \approx \frac{2e^2}{(\hbar^2/GmM^2)} \frac{N}{D} = \frac{2.687 \times 10^{32}}{D}$. This makes $R \to [(2.687 \times 10^{32})/D]^{1/D}$ which goes to unity as $D \to \infty$. This limiting value of R will be explained later.

3. – The extreme relativistic case

We can extend the theory of a free-electron gas to the relativistic domain by replacing the classical kinetic energy with the relativistic formula. In particular, we consider the extreme relativistic case in which the energy is related to the momentum by $E \approx pc = \hbar ck$, and thus eqs. (3) and (5) become

$$N_S = (D-1)\left(\frac{L}{2\pi}\right)^D \frac{\pi^{D/2}}{\Gamma(1+D/2)} (E/\hbar c)^D, \tag{15}$$

$$g(E) = D(D-1)\left(\frac{L}{2\pi}\right)^D \frac{\pi^{D/2} E^{D-1}}{\Gamma(1+D/2)(\hbar c)^D}. \tag{16}$$

As we did in the previous section, the Fermi energy is obtained from the density of states $g(E)$ and the use of eq. (2) gives us

$$E_{\mathrm{F}} = \frac{2\hbar c}{R}\left[\frac{Nq}{(D-1)}\left\{\Gamma\left(1+\frac{D}{2}\right)\right\}^2\right]^{1/D}. \tag{17}$$

The quantum-mechanical energy of the system is

$$E_m = \int_0^{E_\mathrm{F}} E\, g(E)\mathrm{d}E = 2\hbar c \frac{D}{D+1} Nq\sqrt{\pi}\left[\frac{Nq}{V}\frac{\Gamma(1+D/2)}{D-1}\right]^{1/D} \tag{18}$$

$$= 2\hbar c\frac{D}{D+1}\left[\frac{(Nq)^{D+1}}{D-1}\{\Gamma(1+D/2)\}^2\right]^{1/D}\frac{1}{R}.$$

The above equation and eq. (10) give the total energy E whose minimum requires that $\mathrm{d}E/\mathrm{d}R = 0$, which easily yields the radius of stabilization R, namely

$$R^{D-3} = \frac{(D-2)(D+1)GNM^2}{2(D+2)\hbar cq}\left[\frac{Nq}{(D-1)}\{\Gamma(1+D/2)\}^2\right]^{-1/D}. \tag{19}$$

Again, the quantum-mechanical energy, E_m, plays a role in determining the degeneracy pressure, P, so that when the system expands by dV the mechanical energy decreases by dE_m which is given by $\mathrm{d}E_m = -(E_m/DV)\mathrm{d}V$. Comparison of d$E_m$ with the work d$W(= P\mathrm{d}V)$ gives the degeneracy pressure, which, by using eq. (18), takes the form

$$P = \frac{E_m}{DV} = \frac{2\hbar cNq}{(D+1)\pi^{D/2}R^{D+1}}\left[\frac{Nq}{D-1}\right]^{1/D}\left\{\Gamma\left(1+\frac{D}{2}\right)\right\}^{(D+2)/D}. \tag{20}$$

Our results in eqs. (19) and (20) show, respectively, the dependence of R and P on the space dimension D. It is instructive to note that there is no special value for R in the three-dimensional space ($D = 3$). In this particular case, if the total energy is positive, then degeneracy forces exceed gravitational forces and the star will expand, whereas if the total energy is negative, then gravitational forces exceed the degeneracy forces and thus the star will collapse. Therefore, one can find the critical number of nucleons, N_c, by equating the quantum-mechanical energy E_m, given by eq. (18), with the gravitational energy E_g, given by eq. (10) for $D = 3$. The result is

$$N_c = \frac{3}{2}q^2\left[\frac{5\hbar c\pi^{1/3}}{4GM^2}\right]^{3/2}. \tag{21}$$

The above result is the Chandrasekhar limit in the three-dimensional space. Stars with number of nucleons $N > N_c$ will collapse further and will not be halted by the electron degeneracy pressure. As we did in the previous section, and for calculation purposes, we consider a star whose mass equals the mass of the Sun (1.989×10^{30} kg) and thus the number of nucleons $N = 1.188 \times 10^{57}$. Substituting the numerical values of the constants involved in eqs. (19) and (20), we obtain numerical values for the stabilization radius R and the degeneracy pressure P for the star, which we report in table II.

It is instructive to plot graphs that show the trend in R and in P as the dimension D changes. This has been done in fig. 1 and fig. 2, respectively.

Figure 1 shows the relation between $\log R$ and the space dimension D. It is noticed that the radius of stability decreases as the dimension increases, which means that the balance between degeneracy pressure and gravitational force is attained at smaller star size as the dimension increases. Figure 2 shows the relation between $\log P$ and the space dimension D. It is noticed that the degeneracy pressure decreases as D increases up to

TABLE II. – *The stability radius R and the degeneracy pressure P of a white-dwarf star (extreme relativistic).*

D	R	P	D	R	P
4	6.97×10^4	1.56×10^{21}	13	32.14	1.44×10^{14}
5	8.07×10^3	1.20×10^{18}	14	25.08	1.37×10^{14}
6	1.83×10^3	6.09×10^{16}	15	20.23	1.43×10^{14}
7	627.4	8.33×10^{15}	16	16.76	1.62×10^{14}
8	280.8	2.05×10^{15}	18	12.26	2.55×10^{14}
9	150.2	7.56×10^{14}	20	9.53	5.15×10^{14}
10	91.0	3.76×10^{14}	40	3.09	4.13×10^{20}
11	60.40	2.32×10^{14}	60	2.12	2.45×10^{29}
12	42.97	1.67×10^{14}	100	1.57	4.77×10^{50}

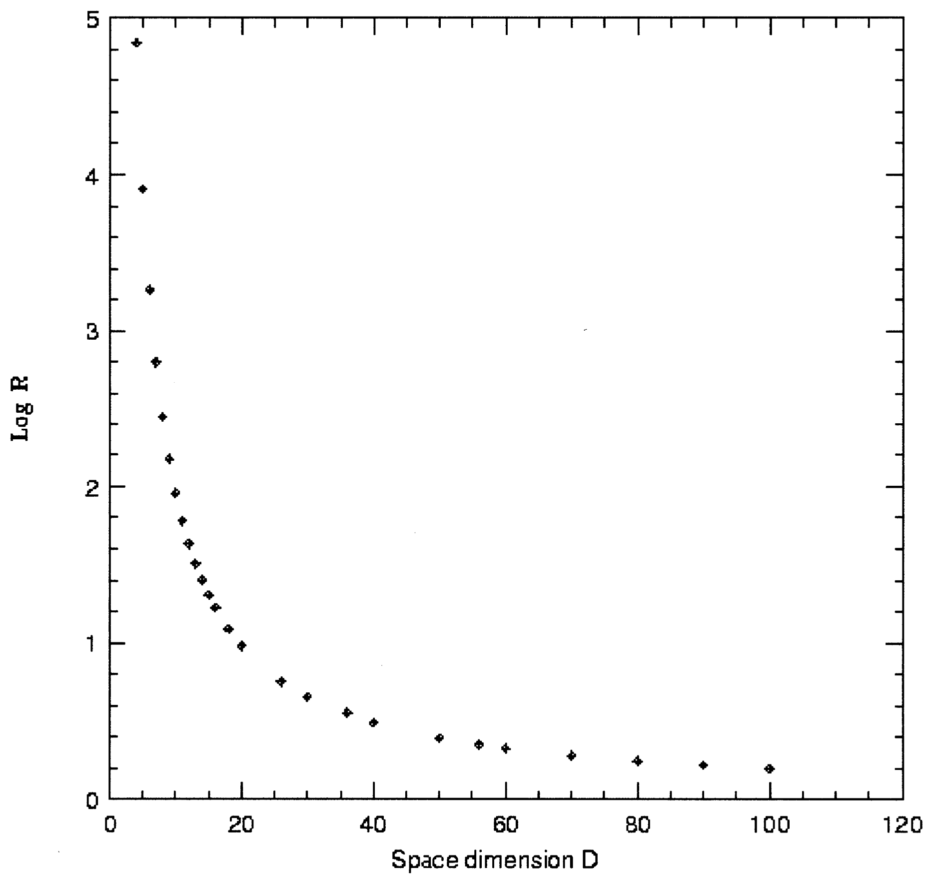

Fig. 1. – Dependence of stability radius R on space dimension D of a white dwarf (extreme relativistic).

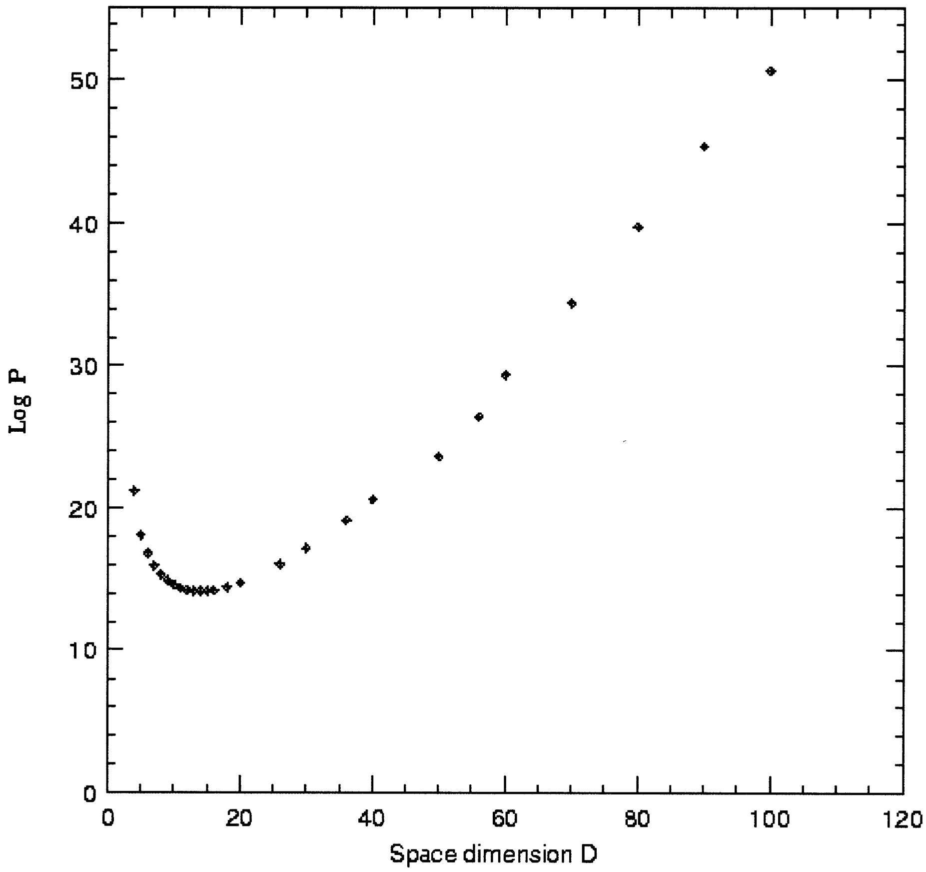

Fig. 2. – Dependence of degeneracy pressure P on space dimension D of a white dwarf (extreme relativistic).

Table III. – *Stability radius R of a neutron star (non-relativistic) in different dimensions.*

D	R	D	R
3	1.24×10^4	12	1.20×10^3
5	2.47×10^{11}	13	6.54×10^2
6	4.04×10^7	14	3.94×10^2
7	9.46×10^5	20	50.74
8	9.73×10^4	40	7.21
9	2.02×10^4	60	4.30
10	6.24×10^3	100	2.17
11	2.50×10^3	10^4	1.007

TABLE IV. – *Stability radius R, Fermi energy E_F, and degeneracy pressure P for a neutron star in different dimensions D (extreme relativistic).*

D	R	E_F	P	D	R	E_F	P
4	2.93×10^4	2.69×10^3	2.82×10^{22}	18	11.67	1.78×10^{-4}	1.30×10^{15}
5	5.32×10^3	23.69	3.34×10^{19}	20	9.13	1.21×10^{-4}	2.62×10^{15}
6	1.40×10^3	1.28	8.92×10^{17}	40	3.03	2.64×10^{-5}	1.88×10^{21}
7	514.68	0.171	8.17×10^{16}	60	2.09	1.90×10^{-5}	1.18×10^{30}
8	240.30	0.039	1.82×10^{16}	80	1.76	1.738×10^{-5}	1.14×10^{40}
9	132.10	0.013	5.90×10^{15}	86	1.69	1.731×10^{-5}	2.57×10^{43}
10	81.60	5.17×10^{-3}	2.68×10^{15}	88	1.67	1.730×10^{-5}	3.45×10^{44}
11	54.95	2.52×10^{-3}	1.55×10^{15}	90	1.65	1.731×10^{-5}	4.91×10^{45}
12	39.53	1.40×10^{-3}	1.05×10^{15}	94	1.62	1.735×10^{-5}	1.02×10^{48}
13	29.83	8.58×10^{-4}	8.64×10^{14}	98	1.58	1.741×10^{-5}	2.33×10^{49}
14	23.44	5.67×10^{-4}	7.93×10^{14}	100	1.56	1.76×10^{-5}	1.83×10^{51}
15	19.02	3.98×10^{-4}	8.03×10^{14}	110	1.51	1.77×10^{-5}	3.67×10^{57}
16	15.84	2.93×10^{-4}	8.83×10^{14}	120	1.45	1.81×10^{-5}	1.09×10^{64}

$D = 14$, but then it increases. It must be emphasized here that the decreasing part of P is compensated by a decrease in the gravitational force which can be simply checked from eq. (10) which yields

$$F_g = -\frac{\mathrm{d}E_g}{\mathrm{d}R} = \frac{D(D-2)}{(D+2)} \frac{GN^2M^2}{R^{(D-1)}} ,$$

and therefore R continues to decrease as D increases.

We note that in the extreme relativistic case the radius of stability R is smaller than its value in the non-relativistic case for any dimension D. The origin of this difference is attributed to the relation between the energy E and the wave vector k, where $E = \hbar^2 k^2/2m$ in the non-relativistic case and $E = \hbar ck$ in the extreme relativistic case. This implies a different energy dependence of the density of states for the two cases as is seen in eqs. (5) and (16). This has a consequence on physical properties of the system. Among these properties, the degeneracy pressure P in the extreme relativistic case is higher than its counterpart in the non-relativistic case. Therefore, the degeneracy pressure in a star in the extreme relativistic case can balance greater gravitational force than in the non-relativistic case, and thus it is expected that the radius of stability R would be smaller in the extreme relativistic case than in the non-relativistic one.

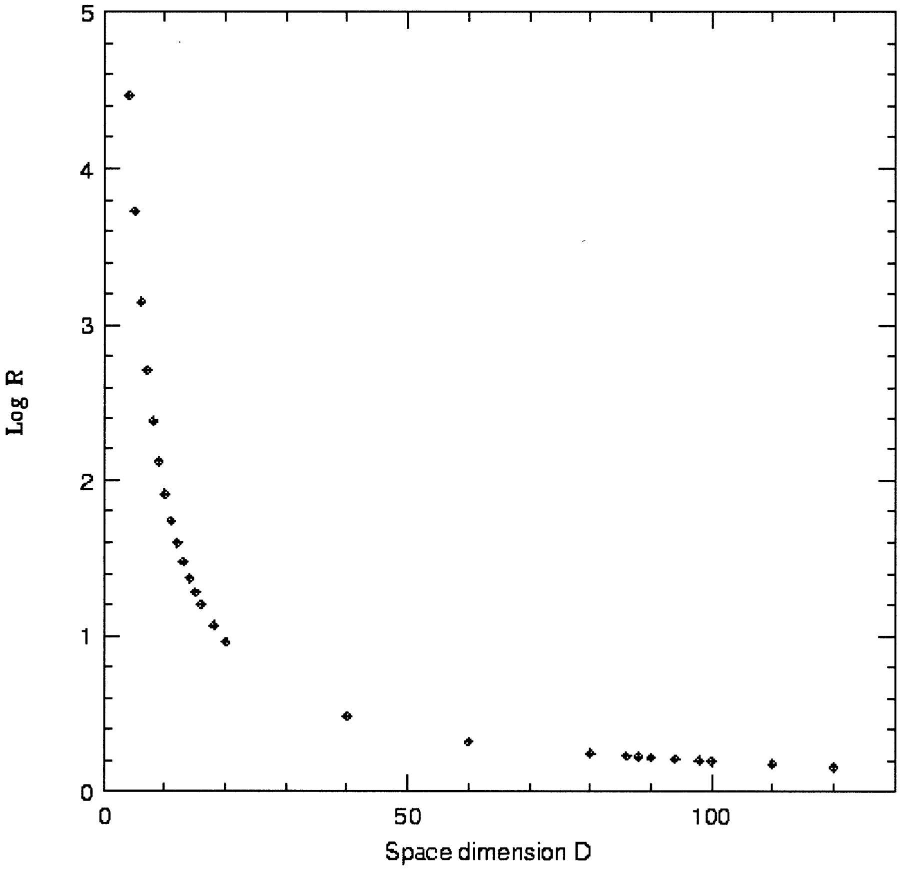

Fig. 3. – Dependence of radius of stability R on space dimensions D of a neutron star (extreme relativistic).

As for the non-relativistic case, Stirling's formula for very large D makes $[\Gamma(1 + D/2)]^{-2/D} \rightarrow (2e/D)$ and thus eq. (19) gives $R \rightarrow (3.4 \times 10^{19})^{1/D}$ which goes to 1 as $D \rightarrow \infty$. We also remark that the electron degeneracy pressure P decreases as the space dimension D increases and reaches a minimum value of 1.37×10^{14} at $D = 14$, beyond which P increases without upper bound so that it goes to infinity as $D \rightarrow \infty$. This demonstrates that the degenerate electrons exert infinite quantum pressure in the infinite-dimensional space. This is consistent with the observation that the limit of the volume of a hyper-sphere as $D \rightarrow \infty$ is zero for $R \rightarrow 1$. The unity limit of the stability radius in the infinite-dimensional space can be analyzed as follows: eq. (10) gives, for large D, the gravitational force $F_g = 2DGN^2M^2/R^{D-1}$, while eq. (18) gives the quantum-mechanical force $F_m = 2\hbar cNqD/R^2$, and thus the ratio F_g/F_m goes to $GNM^2/\hbar cqR^{D-3}$. It is noticed that when $D \rightarrow \infty$, this ratio goes to zero if $R > 1$ and goes to ∞ if $R < 1$.

4. – Neutron degeneracy pressure

When the mass of a star is greater than the Chandrasekhar limit, it collapses further and in this case the Fermi energy of the electrons increases to the point where it is

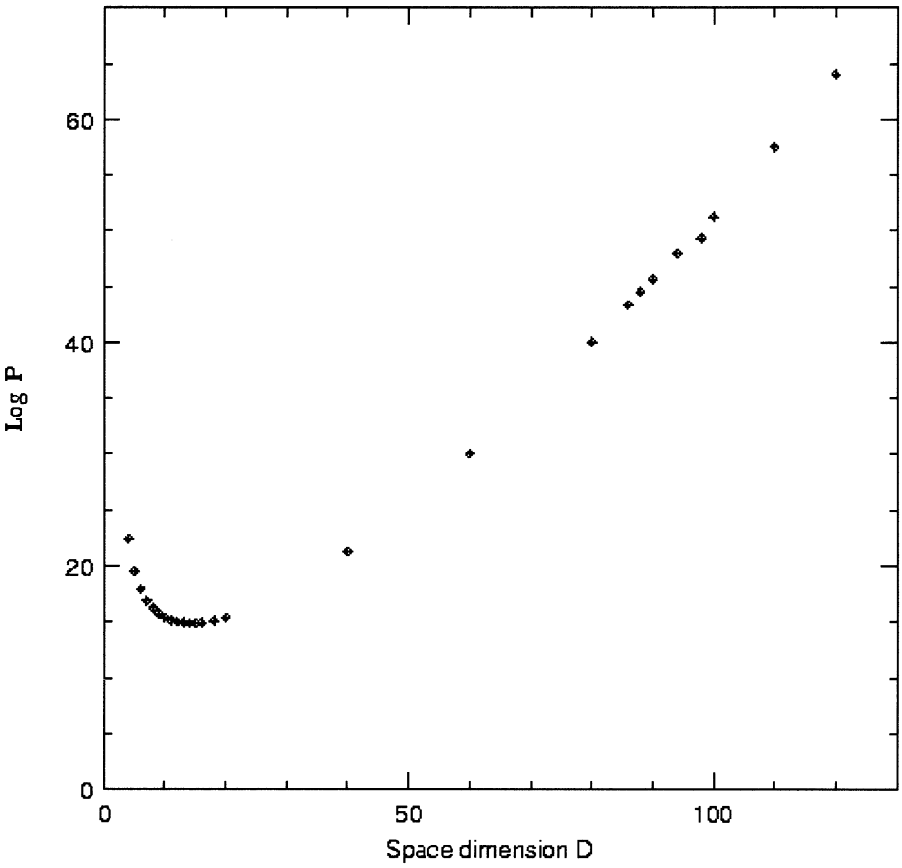

Fig. 4. – Dependence of degeneracy pressure P on space dimension D for a neutron star (extreme relativistic).

energetically favorable for them to combine with the protons to produce neutrons via the inverse beta decay which is called electron capture. The result of this collapse is an extremely compact neutron star. When neutrons are backed together, the number of available low-energy states is too small and many neutrons are forced into high-energy states. These high-energy neutrons make up the entire pressure supporting the neutron star, which is called neutron degeneracy pressure. The radius of stability$R$ of such a neutron star (non-relativistic) can be calculated, for each D, by using eq. (14), except that here the mass m is replaced by the mass of the neutron M and the parameter $q = 1$. We show this in table III, for a star whose mass equals the mass of the Sun.

It is noticed that R has a maximum value when $D = 5$, then it decreases and reaches an asymptotic value of 1 as $D \to \infty$. As is mentioned for the white dwarf, this maximum value has no physical grounds, since the electrons become relativistic. This maximum disappears when the extreme relativistic case is considered.

In the extreme relativistic case, the balancing between the neutron degeneracy forces and the gravitational forces can be discussed. The radius of stability R is computed from eq. (19) with $q = 1$. These values of R can be used in eq. (17) with $q = 1$ to calculate the Fermi energy E_{F}, and furthermore the neutron degeneracy pressure P is readily calculated by using eq. (20). All these results are shown in table IV.

As before, it is interesting to plot log R and log P *vs.* D which is shown in fig. 3 and fig. 4, respectively.

The behavior of R, E_{F}, and P for very large D is obtained using Sirling's formula. As was shown earlier, R approaches 1 as $D \to \infty$. Letting $n = D/2$, we get $[\Gamma(1+D/2)]^{2/D} = [\Gamma(1+n)]^{1/n} \approx (2\pi)^{1/2n} n^{(1+1/2n)} e^{-1} \approx ne^{-1}$, and hence $E_{\mathrm{F}} \approx (\hbar c/e)D$ which goes to ∞ in the infinite-dimensional space ($D = \infty$). The behavior of the neutron degeneracy pressure P with the space dimension D is similar to that of the electron degeneracy pressure. It decreases as D increases and reaches a minimum value of 7.93×10^{14} at $D = 14$ after which it increases without upper bound and becomes infinite as $D \to \infty$, since we have $[\Gamma(1+D/2)]^{(D+2)/D} \approx n^{n+3/2} e^{-n}$, and therefore $P \approx D^D e^{-D}$ which goes to ∞ as $D \to \infty$. It is noticed that the Fermi energy E_{F} decreases as D increases and reaches a minimum value at $D = 88$ beyond which it asymptotically increases as $(\hbar c/e)D$.

5. – Conclusions and results

In the present paper, we considered degenerate electron gas and derived the quantum-mechanical energy, Fermi energy, and the degeneracy pressure in D-dimensional space for the non-relativistic and the extreme relativistic cases. The role of the degeneracy pressure in the balancing against the gravitational forces of the system was examined. For computation purposes, we have applied our results to the stability of white-dwarf and neutron stars against gravitational collapse. The radius of stability R was obtained by requiring that the total energy (quantum-mechanical energy plus the gravitational energy) is a minimum. In the non-relativistic case, it has been found that R has a maximum at a space dimension $D = 5$ and beyond this value it decreases as D increases and approaches unity in the infinite-dimensional space ($D \to \infty$). This maximum has no physical grounds since the electrons become relativistic as can be seen by calculating the Fermi energy. We also noticed that there is no special value for R when $D = 4$. In the extreme relativistic case, we found that there is no special value of R when $D = 3$ in which case our result yields the critical number of nucleons N_C (called Chandrasekhar limit) beyond which gravitational collapse occurs. We also showed that the radius of stability decreases as D increases and again approaches unity in the infinite-dimensional space. Furthermore, we observed that the electron degeneracy pressure P decreases as the dimension D increases and approaches a minimum value at $D = 14$ and then starts to increase with increasing D and becomes infinite in the infinite-dimensional space. It is remarkable to note that the gravitational force approaches ∞, in the infinite-dimensional space, only when $R = 1$. This explains why the radius of stability approaches unity as $D \to \infty$. Finally, we have been able to extend our results to neutron degeneracy pressure. Here, the results showed (in non-relativistic case) that R has a maximum value at $D = 5$ (which again has no physical ground) and goes to 1 as $D \to \infty$. For the extreme relativistic case, we calculated the radius of stability of a neutron star, the Fermi energy, and the neutron degeneracy pressure. In this case we noticed that for $D = 3$ there is no special value for R that minimizes the total energy. It was found that both the radius of stability and the neutron degeneracy pressure have similar behaviors as those for the electron degenerate system. It has been found that the stability radius for the neutron star is smaller than its value in the white dwarf in all space dimensions. Concerning the Fermi energy, table IV shows that E_{F} decreases as D increases and reaches a minimum value at $D = 88$ and then increases to the asymptotic formula $E_{\mathrm{F}} \approx (\hbar c/e)D$. The present work may shed some light on the role of large extra dimensions and brane physics in modern physics theories.

REFERENCES

[1] Belhaj A. and Rasneusser J., *J. Math. Phys.*, **46** (2005) 043511.
[2] Golovnev V., *J. Math. Phys.*, **47** (2006) 082105.
[3] Dong S. and Sun G., *Physica Scripta*, **70** (2004) 9497.
[4] Lin D. H., *J. Phys. A: Math. Gen.*, **30** (1997) 3201.
[5] Znojil M., *J. Phys. A: Math. Gen.*, **33** (2000) 4203.
[6] Romera E., Moreno P. and Dehesa S., *J. Math. Phys.*, **4** (2006) 103504.
[7] Katatheh Q., Hall R. and Saad N., *J. Phys. A: Math. Gen.*, **40** (2007) 13431.
[8] Bender C., Boettcher S. and Lipatov L., *Phys. Rev. D*, **46** (1992) 5557.
[9] Bender C. and Boettcher S., *Phys. Rev. D*, **51** (1994) 1875.
[10] Troncoso R. and Zanelli J., *Class. Quantum Grav.*, **17** (2000) 4451.
[11] Tanabe M. and Maede K., *J. Phys.: Conf. Ser.*, **31** (2006) 215.
[12] Gibbons W., Lu H., Page N. and Pope C., *Phys. Rev. Lett.*, **93** (2004) 171102.
[13] Rogatko M., *Class. Quantum Grav.*, **19** (2002) L151.
[14] Berti E., Cardeso V. and Casals M., *Phys. Rev. D*, **73** (2006) 024013.
[15] Kunduti H., Lucietti J. and Reall S., *Phys. Rev. D*, **74** (2006) 084021.
[16] Ujjal D. and Subenoy C., *Gen. Relativ. Quantum Cosmol.*, **36** (2004) 1243.
[17] Gaswami R. and Joshi P., *Phys. Rev. D*, **69** (2004) 044002.
[18] Ryttov T. and Sannino F., *Phys. Rev. D*, **76** (2007) 105004.
[19] Gaswami R. and Joshi P., *Phys. Rev. D*, **76** (2007) 084026.
[20] Krtous P., *Phys. Rev. D*, **76** (2007) 084035.
[21] Chavanis P., *Int. J. Mod. Phys. B*, **20** (2006) 3113.
[22] Bandyopadhyay T. and Chakraborty S., *Mod. Phys. Lett. A*, **22** (2007) 2839.
[23] Ahmad S., Khan S. and Nansoor S., *Nanotechnology*, **17** (2006) 1686.
[24] Stewart J., *Class. Quantum Grav.*, **17** (2000) 513.
[25] Zhou L., Ma C. and Ma Y., *J. Phys. B: At. Mol. Opt. Phys.*, **40** (2007) 2591.
[26] Silverman M., *Int. J. Mod. Phys. D*, **15** (2006) 2257.
[27] Menon V. and Agrawal D., *J. Phys. A: Math. Gen.*, **31** (1998) 1109.
[28] Bender C. M., *Am. J. Phys.*, **63** (1995) 101.
[29] Griffiths D. J., *Introduction to Quantum Mechanics* (Prentice-Hall, New Jersey) 1995.